



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BHAM	LEVEL: 8
COURSE CODE: ANA801S	COURSE NAME: APPLIED NUMERICAL ANALYSIS
SESSION: JULY 2022	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 120 (to be converted to 100%)

2ND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER

EXAMINERS	PROF S. A. REJU
MODERATOR:	PROF S. MOTSA

INSTRUCTIONS

1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencils.
3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [30 MARKS]

Discuss exhaustively the Romberg Method Extrapolation process to show that the n th order extrapolation employed by the method is given by:

$$I_{Improved} = \frac{4^n I_{More-accurate} - I_{Less-accurate}}{4^n - 1}$$

QUESTION 2 [30 MARKS]

(a) Define the Picard Method for solving the following Initial Value Problem (IVP)

$$\frac{dy}{dt} = y'(t) = f(t, y(t)), y(t_0) = y_0$$

and hence derive the Picard Iteration algorithm [13]

(b) Using the Picard method, find the solution, correct to 3 decimal places, of the following 1st order IVP at $x = 0.1$

$$\frac{dy}{dx} = x + y^2, y(0) = 1$$

with $x(0) = x_0 = 0$ [17]

QUESTION 3 [30 MARKS]

(a) Discuss the contrast between a quadrature rule and the adaptive rule. [3]

(b) Consider the integral [27]

$$\int_a^b f(x) dx = \int_1^3 e^{2x} \sin(3x) dx$$

Using the Adaptive Simpson's Method and an error $\epsilon = 0.2$, obtain the approximate value of the above integral (for computational ease, using where appropriate the following as done in class):

$$\frac{1}{10} \left| S(a, b) - S\left(a, \frac{a+b}{2}\right) - S\left(\frac{a+b}{2}, b\right) \right|$$

where

$$\int_a^b f(x) dx = (S(a, b) - \frac{h^5}{90} f^{(4)}(\xi)), \quad \xi \in (a, b)$$

QUESTION 4 [30 MARKS]

- (a) (i) State the Steepest Descent Algorithm [6]
(ii) State the theorem that guarantees that the Steepest Descent method ensures some progress in the direction of the minimum of the objective function during each iteration. [4]

- (b) Using the Steepest Descent Method, obtain the minimum of the following function:

$$f(x, y) = 4x^2 - 4xy + 2y^2$$

[20]

END OF QUESTION PAPER

TOTAL MARKS = 120